B.Sc. Part-I

Paper-I

Theory of Relativity

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Lorentz Transformation

Date 10.6

Expt. No. _____ Page No. _____

The Lorentz transformations are a six-parameter family of linear transformations from a coordinate frame in spacetime to another frame that moves at a constant velocity relative to the former. The respective inverse transformation is then parameterized by the −ve of this velocity. The transformations are named after the Dutch Physicist Hendrik Lorentz.

The most common form of the transformation, parameterized by the real constant v, representing a velocity confined to the x-direction, is expressed as −

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$
$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$

where (t, x, y, z) and (t', x', y', z') are the coordinates of an event to two frames, where the primed frame is seen from the unprimed frame as moving with speed v along the x-axis, c is the speed of light, and $\gamma = \left(\sqrt{1 - \frac{v^2}{c^2}}\right)^{-1}$ is the Lorentz factor. When speed v is much smaller than c, the Lorentz factor is negligibly different from 1, but as v approaches c, γ grows without bound. The value of v must be smaller than c for the transformation to make sense.

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Expressing the speed as $\beta = v/c$, an equivalent form of the transformation is

$$ct' = \gamma(ct - \beta x)$$
$$x' = \gamma(x - \beta ct)$$
$$y' = y$$
$$z' = z,$$

Frames of reference can be divided into two groups. Inertial and non-inertial (accelerating moving in curved paths, rotational motion with constant angular velocity etc.). The term Lorentz transformations" only refers to transformations between inertial frames, usually in the context of special relativity.

In each reference frame an observer can use a local coordinate system to measure lengths, and a clock to measure time intervals. An event is something that happens at a point in space at an instant of time, or more formally a point in spacetime. The transformations connect the space and time coordinates of an event as measured by an observer in each frame.

They supersede the Galilean transformation of Newtonian physics, which assumes an absolute space and time. The Galilean transformation ~~of Newtonian physics, which a~~ is a good approximation only at relative speeds much less than the speed of light. Lorentz transformations have a no. of unintuitive features that do not appear in Galilean transformations. for example, they reflect the fact that observers

moving at different velocities may measure different distances, elapsed times and even different orderings of events, but always such that the speed of light is the same in all inertial reference frames. The invariance of light speed is one of the postulates of special relativity.

The transformations were the result of attempts by Lorentz and others to explain how the speed of light was observed to be independent of the reference frame and to understand the symmetries of the laws of electromagnetism. The Lorentz transformation is in accordance with Special relativity, but was derived first.

The Lorentz transformation is a linear transformation. It may include a rotation of space, a rotation-free Lorentz transformation is called a Lorentz boost. In the Mathematical model of spacetime in special relativity - the Lorentz transformations preserve the spacetime interval between any two events. This property is the defining property of a Lorentz transformation. They describe only the transformations in which the spacetime event at the origin is left fixed. They can be considered as a hyperbolic rotation of Minkwski space. The more general set of transformations that also includes translations is known as the Poincare group.